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[ijirset@gmail.com](mailto:ijirset@gmail.com)

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# Determination of unknowns in Electrical Circuits using Linear Equations and Matrices

Parshuram A.Ahire

Department of Mathematics, M.S.G College Malegaon-Camp, Dist- Nasik, Maharashtra, India

**ABSTRACT:** This paper is useful to explain simple ideas to solving problems involving electrical circuits in the general format. In a electrical circuit, if batteries and resistors are connected by wires, determine all unknown values of electrical current flowing through the wires, potential difference across the batteries, and resistance in the resistors, given enough known values of same quantities.

**KEYWORDS:** Wire, Battery, Resistor, Current node, Voltage loop, Kirchhoff's voltage law, Kirchhoff's current law.

## I. INTRODUCTION

In this paper we introduce the basic laws of electric circuits and then use to analyze electric-circuits consisting of batteries, resistors and wires.

A source of direct current(or voltage) in the circuit is called a battery, A device, such as a light bulb, that reduces the current in circuit by converting electrical energy into thermal energy is called a resistor, and a conductor that allows a free flow of electric current is called a wire. A closed connection of resistors, batteries, and wires is known as simple electrical circuit. When circuits are represented by diagrams, batteries, resistors, and wires are depicted as follows:

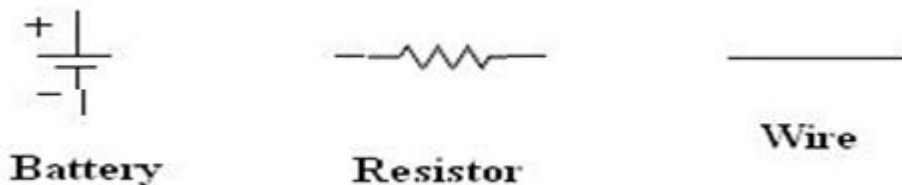


Figure 1 shows a simple electrical circuit consisting of three batteries and four resistors connected by wires.

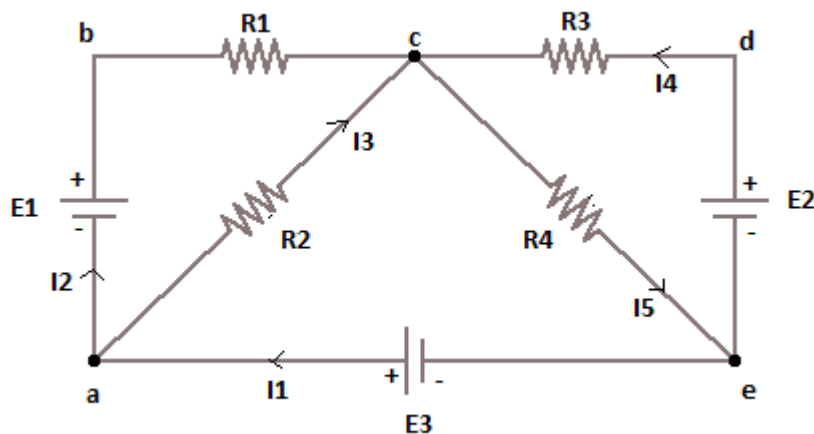


Figure 1



## II. BASIC DEFINITIONS

The physical quantities used when discussing electrical circuits are current, resistance, and electrical potential difference across a battery. Electrical potential difference is denoted by  $E$  and is measured in volts (V), Current is denoted by  $I$  and is measured in amperes (A), Resistance is denoted by  $R$  and is measured in ohms( $\Omega$ ). These units are all related by the equation

$$1 \text{ Volt} = 1 \text{ Amp.} \times 1 \text{ Ohm.}$$

The electrical potential difference of a battery is taken as positive when measured from the negative terminal (-) to the positive terminal (+), and negative when measured from the positive terminal (+) to negative terminal (-). The electrical potential difference across a resistor (denoted by  $V$ ), which depends on the current flowing through the resistor ( $I$ ) and its resistance ( $R$ ), then Ohms law states that:

$$V = \pm I R.$$

The negative (-) sign is used when the difference is measured across the resistor in the direction of the current flow, while the positive (+) sign is used when the difference is measured across the resistor in direction opposite the current flow.

All electrical circuits consist of current nodes and voltage loops. In electrical circuit, a point where three or more segments of wire meet is called **current node**. For example, **Figure 1** contains three current nodes at points **a**, **c** and **e**.

Points **b**, and **d** are not current nodes since only two wire segments meet at these points.

A closed connection within the circuit is called a **voltage loop**, For example, Figure 1 contains the six voltage loops.viz

$$\begin{aligned} & a \rightarrow b \rightarrow c \rightarrow a, \\ & c \rightarrow d \rightarrow e \rightarrow c, \\ & a \rightarrow c \rightarrow e \rightarrow a. \\ & a \rightarrow b \rightarrow c \rightarrow e \rightarrow a. \\ & \quad a \rightarrow c \rightarrow d \rightarrow e \rightarrow a. \\ & \text{and} \quad a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow a. \end{aligned}$$

## III. BASIC LAWS

The physical laws that govern the flow of current in an electrical circuit are conservation of charge and conservation of energy.

- **Kirchhoff's current law** (The conservation of charge) :At any current node, the flow of the current into the node equals the flow of the current outof the node. This guarantees that no charge builds up or is depleted at a node, so the current flow is steady through the node.
- **Kirchhoff's voltage law** (The conservation of energy) :Around any voltage loop, the total electrical potential difference is zero.

We are now ready to use these ideas in physical laws and the methods for solving linear systems (i.e. Gauss Elimination Method or Gauss-Jordon Elimination Method) to solving problems involving electrical circuits, which have the following general format. In a circuit containing batteries, resistors, and wires, determine all unknown values of aelectrical potential difference across the batteries, resistance in the resistors, and current flowing through the wires, given enough known values of these same quantities. We illustrate this bythe following example for the standard situation when the unknowns are the currents.

-----**Example 1.**Figure 2 shows the circuit in**Figure 1** with assigned values of electrical potentials to the batteries, measured from the negative terminal to the positive one, and the indicated resistances to the resistors. The problem is to determine the currents that flow through each segment of the circuit.

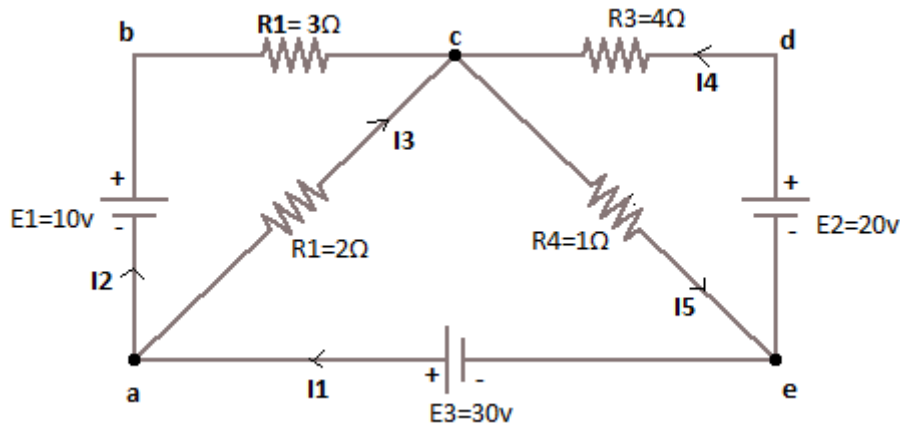


Figure 2

We begin by assigning currents to each segment of the circuit that begins at some node point and ends at some other node point (with no other node points in between). For example, in **Figure 2**, we assign  $I_1$  to the segment  $e \rightarrow a$ ,  $I_2$  to the segment  $a \rightarrow b \rightarrow c$ ,  $I_3$  to the segment  $a \rightarrow c$ ,  $I_4$  to the segment  $e \rightarrow d \rightarrow c$ , and  $I_5$  to the segment  $c \rightarrow e$ . In addition, we arbitrarily assign directions to these currents as indicated by the arrows in **Figure 2**. If the assigned direction is correct, the computed value of the current will be positive, while if the assigned direction is incorrect, the computed value of the current will be negative. The latter situation indicates that the actual direction of current flow is just opposite to assigned one. Using Kirchhoff’s current law (the sum of the incoming currents = the sum of the outgoing currents) at points **a**, **c** and **e**, we have

At **a**,  $I_1 = I_2 + I_3$

At **c**,  $I_2 + I_3 + I_4 = I_5$   
 and at **e**,  $I_5 = I_1 + I_4$  respectively.

Since second equation contains the same information as in first and third equations (i.e. second equation is a linear combination of the other two equations), only two of them are needed

$$I_1 = I_2 + I_3 \text{ ..... (1)}$$

$$\text{and } I_5 = I_1 + I_4 \text{ ..... (2)}$$

**In general, if circuit has n nodes, Kirchhoff’s current law will yield n – 1 useful equations and one equation that is a linear combination of the other n – 1 equations.**

Now we use **Kirchhoff’s** voltage law. Starting at point **a** and moving through the battery from ( - ) to ( + ) to point **b**, the potential is **+10 V**. Moving from point **b** to point **c** through the **3Ω** resistor results in a potential difference of **-3I<sub>2</sub>**. Moving from point **c** to point **a** through the **2Ω** resistor results in potential difference of **+2I<sub>3</sub>** (across the resistor). To summarize, applying Kirchhoff’s voltage law around the closed loop **a → b → c → a** leads to

$$(+E_1) + (-R_1 I_2) + (+R_2 I_3) = 0$$

$$\text{or } (+10) + (-3I_2) + (+2I_3) = 0$$

Which simplifies to

$$3I_2 - 2 I_3 = 10 \text{ ..... (3)}$$

In a similar way, applying Kirchhoff’s voltage law around the closed loop **c → d → e → c** leads to

$$(+R_3 I_4) + (-E_2) + (+R_4 I_5) = 0$$

$$\text{or } 4I_4 - 20 + 1.I_5 = 0$$

$$\text{This simplifies to } 4 I_4 + I_5 = 20. \text{ ..... (4)}$$



Also, around the closed loops  $a \rightarrow c \rightarrow e \rightarrow a$ , we get

$$2I_3 + I_5 - 30 = 0 \rightarrow 2I_3 + I_5 = 30 \text{ -----} \tag{5}$$

Again applying Kirchhoff's voltage law around the closed loop  $a \rightarrow b \rightarrow c \rightarrow e \rightarrow a$  leads to

$$(+E_1) + (-R_1I_2) + (-R_4I_5) + (+E_3) = 0$$

$$\text{Or } (+10) + (-3I_2) + (-1I_5) + 30 = 0 \text{ gives } 3I_2 + I_5 = 40 \text{ -----} \tag{6}$$

Similarly, around the closed loops  $a \rightarrow c \rightarrow d \rightarrow e \rightarrow a$ , and

$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow a$ . we get

$$2I_3 - 4I_4 + 20 - 30 = 0 \rightarrow I_3 - 2I_4 = 5 \text{-----} \tag{7}$$

$$\text{and } 10 - 3I_2 + 4I_4 - 20 + 30 = 0 \rightarrow 3I_2 - 4I_4 = 20 \text{ -----} \tag{8}$$

But equation (8) is eq.(6) - eq.(4), eq.(7) is eq.(5) - eq.(4), and eq.(6) is eq.(5) + eq.(3), and it therefore provides no new information. Thus this equation is redundant and can be omitted.

In general, larger outer loops like  $a \rightarrow b \rightarrow c \rightarrow e \rightarrow a$ ,  $a \rightarrow c \rightarrow d \rightarrow e \rightarrow a$  and  $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow a$ . provides no new information, if all its inner loops, like  $a \rightarrow b \rightarrow c \rightarrow a$ ,  $c \rightarrow d \rightarrow e \rightarrow c$ , and  $a \rightarrow c \rightarrow e \rightarrow a$ . have already been included. Equations (1) to (5), lead to the linear system (In matrix form) as

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 20 \\ 30 \end{bmatrix}$$

Solving this system of linear equations in Matrix form by Gauss Elimination Method or by Gauss-Jordan Elimination Method for  $I_1, I_2, I_3, I_4$ , and  $I_5$  yields (verify)

$$I_1 = 15 \text{ Amp}, I_2 = 8 \text{ A}, I_3 = 7 \text{ A}, I_4 = 1 \text{ A}, \text{ and } I_5 = 16 \text{ A}.$$

[Note: If there is any negative value of  $I$  indicates that its true direction is opposite that as signed in Figure 2].

#### IV. CONCLUSION

In general, for an electrical circuit consisting of batteries, resistors, and wires and having  $n$  different current assignments, Kirchhoff's voltage and current laws will always lead to  $n$  linear equations that have unique solution.

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